# Wavefield Decomposition for Cross-line Survey

The present invention generally relates to apparatus for and
methods of processing seismic data.  It particularly relates
5    to methods of performing a decomposition of a seismic
wavefield into components such as up- and downgoing
wavefield constituents, shear (S) and compressional (P)
waves and/or other constituents of interest, where the
wavefield is obtained through a cross-line survey.
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## BACKGROUND OF THE INVENTION

In the field of seismic exploration, the earth interior is
explored by emitting low-frequency, generally from 0Hz to
200 Hz, acoustic waves generated by seismic sources.
15    Refractions or reflections of the emitted waves by features
in subsurface are recorded by seismic receivers.  The
receiver recordings are digitized for processing.  The
processing of the digitized seismic data is an evolved
technology including various sub-processes such as noise
20    removal and corrections to determine the location and
geometry of the features which perturbed the emitted wave to
cause reflection or refraction.  The result of the
processing is an acoustic map of the earth interior, which
in turn can be exploited to identify for example hydrocarbon
25    reservoirs or monitor changes in such reservoirs.

Seismic surveys are performed on land, in transition zones
and in a marine environment.  In the marine environment,
surveys include sources and receiver cables (streamers)
30    towed in the body of water and ocean bottom surveys in which
at least one of sources or receivers are located at the

seafloor. Seismic sources and/or receivers can also be
placed into boreholes.

The known seismic sources include impulse sources, such as
5    explosives and airguns, and vibratory sources which emit
waves with a more controllable amplitude and frequency
spectrum.  The existing receivers fall broadly speaking into
two categories termed "geophones" and "hydrophones",
respectively.  Hydrophones record pressure changes, whereas
10   geophones are responsive to particle velocity or
acceleration.  Geophones can recorded waves in up to three
spatial directions and are accordingly referred to as 1C, 2C
or 3C sensors. A 4C seismic sensor would be a combination of
a 3C geophone with a hydrophone.  Both types of receivers
15   can be deployed as cables with the cable providing a
structure for mounting receivers and signal transmission to
a base station.

The spatial distribution of source and receiver locations in
20   a seismic survey is referred to as layout or spread. A
variety of spreads are known.  Among those are spreads where
receiver lines, a one-dimensional array of receiver
locations, and source lines, the corresponding array of
source or shot locations, are laid out at an angle. For the
25   purpose of this invention, such layouts are referred to as
"cross-line" geometry or acquisition. Such acquisitions have
been described for example by G.L.O Vermeer, in "3D
Symmetric Sampling", 64th Ann. Internat. Mtg: Soc. of Expl.
Geophys. (1994), 906-909 and later in the United States
30   patent no. 6,026,058.

Seismic energy acquired at a receiver may contain upwardly
and/or downwardly propagating seismic energy depending on
the location of the receiver and on the event.  For example
seismic energy when it is incident (travelling upwardly) on
5    the water-seabed interface, be partly transmitted into the
water column and partially reflected back into the seabed.
Thus, a seismic event will consist purely of upwardly
propagating seismic energy above the seafloor, but will
contain both upwardly and downwardly propagating seismic
10   energy below the seafloor.  As another example, seismic
energy when incident on the water-air interface at sea level
will be reflected back into the water column generating so-
called "ghost" events.  It is therefore often of interest to
decompose the seismic data acquired at the receiver into an
15   up-going constituent and a down-going constituent.

Various filters that enable decomposition of seismic data
into up-going and down-going constituents have been
proposed.  For example in "Application of Two-Step
20   Decomposition to Multi-Component Ocean-Bottom Data: Theory
and Case Study", J. Seism. Expl. Vol. 8, 261-278 (1999),
K.M. Schalkwijk et al have suggested that the down-going and
up-going constituents of the pressure just above the
seafloor may be expressed as:

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[1]

$$P^-(f, k_x, k_y) = \frac{1}{2} P(f, k_x, k_y) - \frac{\rho}{2q(f, k)} v_z(f, k_x, k_y),$$

$$P^+(f, k_x, k_y) = \frac{1}{2} P(f, k_x, k_y) + \frac{\rho}{2q(f, k_x, k_y)} v_z(f, k_x, k_y),$$

where P is the pressure acquired at the receiver, $P^-$ is the

up-going constituent of the pressure above the seafloor, $P^+$
is the down-going constituent of the pressure above the
seafloor, f is the frequency, $k_x$, $k_y$ are the horizontal
wavenumbers, $v_z$ is the vertical particle velocity component

5    acquired at the receiver, $\rho$ is the density of the water, and
q is the vertical slowness in the water layer.

As can be seen, the expressions in equation [1] require two
of the components of seismic data recorded at the receiver

10   to be combined.  These expressions are examples of combining
two components of the acquired seismic data.  It may also be
necessary to combine two or more components of the acquired
seismic data in order to decompose the acquired seismic data
into P-wave and S-wave components, or to remove water level

15   multiple events from the seismic data.

Further separation methods including free-surface multiple
removal above the seafloor, wavefield decomposition into up-
and downgoing constituents or P/S events above and below the

20   surface, the splitting of particle velocities and traction
are described in a number of published documents.

In United States patent no. 6,101,408, the ocean bottom
wavefield separation described in three dimensions using an

25   analytical solution.  However, for practical applications,
the filter is reduced to one dimension.  A number of
decomposition equations for various separations are
developed by Amundsen et al. in the above cited United
States patent no. 6,101,408 and in: "Multiple attenuation

30   and P/S splitting of multicomponent OBC data at a
heterogeneous sea floor", Wave Motion 32 (2000), 67-78.  A
further review of decomposition methods for use in

connection with the present invention is presented by L.
Amundsen in: "Elimination of free-surface related multiples
without need of the source wavelet", Geophysics, Vol. 66,
No. 1 (Jan-Feb 2001), 327-341.

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Approximated compact spatial filters are further described
by Osen et al. in: Towards Optimal Spatial Filters for
Multiple Attenuation and P/S-Splitting of OBC Data", EAGE
60th conference, Leipzig, Germany, 8-12 June 1998, 1-29

10    Geophysical Division.  A short length filter is obtained in
terms of powers of $k_x$ using a series expansion.

When applying three-dimensional (3D) wavefield decomposition
methods to data acquired in a cross-line geometry and sorted

15    into 1-fold bins of common mid-points (CMPs) distributed
evenly in a finely spaced "carpet" determined by in-line
source and receiver spacings as proposed by Vermeer, it was
noted that the known filter introduce an unacceptable level
of noise due to sensor variations, statics and other

20    perturbations.

In the light of the above prior art, it is seen as an object
of the present invention to provide filters applicable to
cross-line acquisitions or data collected through cross-line

25    acquisitions and methods of applying such filters.

SUMMARY OF THE INVENTION

According to a first aspect of the invention there is

30    provided a method of decomposing a seismic wavefield,
wherein a 3D wavefield is obtained by a cross-line
acquisition and filtered applying a decomposition filter

having two spatial components or filtering in two spatial
directions to obtain a decomposed representation of the
wavefield.

5    A 3D wavefield for the purpose of this invention involve
obtaining data or time series of measured parameters over an
area. Hence, such data are acquired as series of ideally
closely spaced parallel lines.  The parameters measured are
preferable velocity and pressure data.

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In the cross-line acquisition shot lines and receiver lines
enclose an angle, which is preferably around 90 degrees.

The method of this invention can be applied to any of the
15   existing decomposition equations that include a filter term
depending on the vertical wavenumber $k_z$. Such decompositions
preferable include up- / down going decomposition, P/S
decomposition, elastic decomposition and acoustic
decomposition.

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The filter of the present invention has two spatial
components that when represented in an analytical form are
written as $k_x$ and $k_y$, or as spatial derivatives in x and y,
respectively.  When implemented as machine program these
25   filters are approximated by finite differences.

The filter is preferably a cascaded filter of 1D spatial
filters that are applied sequentially.

30   The filter is preferably a compact filter having a finite
length or support in in-line and cross-line direction. When
analytically derived, wavefield decomposition filters have

infinite extent or support in space and in time. The filter operations generally assume stationary medium properties or in the case of deghosting a locally flat sea surface (both in time and space). The main advantage of introducing

5   compactness is to ensure that medium properties (or sea surface variations) are constant across the aperture (or two dimensional support) of the filter. For the seabed wavefield decomposition filters the full analytical expression of the filter can be written in the frequency

10  wavenumber domain for instance. The infinite support in time can be maintained since medium properties do not vary with time. However, in the spatial directions Taylor expansions of the filter into factors $k_x$, $k_x{}^2$, $k_x{}^3$, ..., $k_y$, $k_y{}^2$, $k_y{}^3$, ... are proposed. When going back to the

15  spatial domain each factor $k_x$ or $k_y$ or its powers simply correspond to a derivative in the x- and y-directions respectively. Spatial derivatives can in turn be implemented with compact local support using 2-point, 3-point, 5-point, or more extended FD approximations.

20  However, there are also other ways of designing compact filters without necessarily relating them to spatial derivatives.

Preferably the spatial filter of the present invention is

25  applied exclusively to the measured pressure wavefield $P(f, k_x, k_y)$. The pressure measurement is usually less sensitive to mismatch in the response of the various receivers used to record the wavefield.

30  It is furthermore advantageous to use a calibration for matching geophone recordings with hydrophone recordings prior to the decomposition filtering, particularly in case

the filter operates on the particle velocity $(v_x, v_y, v_z)$

After the decomposition filter is applied it is possible to
remove multiples or proceed with other known steps to obtain
5      an image of sub-surface, including migration and other
methods known in the art.

These and other aspects of the invention will be apparent
from the following detailed description of non-limitative
10     examples and drawings.

## BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1A    illustrates an ocean bottom acquisition of a 3D
            seismic wavefield using an OBC and a source towed
15               by a seismic vessel;

FIG. 1B    illustrates the spread of data point after shooting
            a single shot line of the acquisition of FIG. 1A;

20     FIG. 1C    illustrates the acquisition of 3D seismic wavefield
            using streamers towed by a first vessel and a
            source towed by a second seismic vessel;

FIG. 2 is a diagram illustrating steps in accordance with
25               an example of the invention; and

FIG. 3 compares the performance of filter approximations s
            in accordance with examples of the invention.

30                                     EXAMPLES
In FIG.1A there is illustrated an example of a seismic
survey in cross-line geometry.  The survey is a marine

seismic survey in a body of water **10** between a seafloor **101**
and a sea surface **102**.  A receiver cable **11** with a plurality
of receivers **111** is laid out on the seafloor **101**.  The
receivers **111** are preferably 4C sensors, though, as will be

5   apparent from the following description, the sensors may
comprise a 1C geophone and a hydrophone only, being thus
capable of recording at least the vertical component of
velocity and pressure at seafloor level.

10  A seismic vessel **12** tows a marine seismic source **13** close to
the sea surface **102**.  The airgun **13** emits at precisely
determined time intervals an impulse of acoustic energy
referred to as "shot".  A dashed line **132** indicates the path
of the towed airgun **13**. The projection **133** of the dashed

15  line **132** onto the seafloor **102** intersects the receiver line
**112** at approximately 90 degrees.  Though it is preferable to
aim for a near-orthogonal orientation of receiver lines to
shot lines, deviations are inevitable under real survey
conditions.  To facilitate the following description the

20  receiver line or in-line direction is denoted as x
direction, the shot-line or cross-line direction is marked
as y direction and the vertical direction is taken as the z
direction.

25  During a survey, the sources **131** are fired at intervals and
the receivers **121** "listen" within a frequency and time
window for acoustic signals such as reflected and/or
refracted signals that are caused features in path of the
emitted wavefield.  After shooting a line, the vessel

30  performs a u-turn in order to shoot a subsequent line
usually with an offset in receiver line or x direction.

In the general practice, it is assumed that Green's
functions which describe the wave propagation between source
and receiver points are invariant for translation of the
source and receiver in the cross-line direction.  Hence, an
5      offset between shooting lines can be regarded as an equal
shift of the receiver line. As a result, data points
obtained from a single source cross-line form a carpet on
the seafloor, which is illustrated in FIG. 1B. In FIG. 1B,
the triangles **122** denote the location of data points.  As
10     the wavefield is recorded in two spatial dimensions and in
time, the resulting data are referred to as 3D wavefield.

In FIG. 1C, there is shown a schematic cross-line marine
survey with two vessels.  A first vessel **15** tows five
15     streamers **151** below the sea surface following a path **152**.
Simultaneously, a second vessel **16** on path **162** tows a
seismic source **161** below the surface.  As above in Fig. 1A,
the resulting shot and receiver lines are essentially
orthogonal to each other.
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In the following description and the accompanying FIG. 2,
steps are described leading to a decomposition of the 3D
wavefield into up- and downgoing components.

25     After obtaining **20** the wavefield data set as acquired
through the seismic receivers, the data are first preferably
calibrated **21** to compensate for the differences between
geophone and hydrophone recordings.  Any suitable
calibration may be used including for example the methods
30     described in the International patent application
PCT/GB03/04190.  Following those methods, the calibration
can be done using an in-line shot-line for the P, $v_z$ and $v_z$

components and using a cross-line shot-line for the $v_y$ component.

Acoustic wavefield decomposition **22** is usually carried out on the pressure component P (involving spatial filtering of $v_z$). Instead in this example decomposition filters are applied to the vertical geophone component $v_z$ (involving spatial filtering of P). The advantage of this example is that the spatial components of the filter only act on P as shown in PCT/GB03/04190.

Accordingly, acoustic wavefield decomposition into up- and down-going constituents above the seafloor can be achieved by solving the following equation:

[2]

$$v_z^{\pm}(f, k_x, k_y) = \frac{1}{2} a(f) \ v_z(f, k_x, k_y) \pm \frac{k_z(f, k_x, k_y)}{2\rho\omega} \ P(f, k_x, k_y)$$

In equation [2], a(f) denotes the optional frequency-dependent calibration filter that corrects for imperfections in the recording of the geophones, $v_z^-$ denotes the up-going constituent of the vertical component of particle velocity, and $v_z^+$ denotes the down-going constituent of the vertical component of particle velocity. The velocity $v_z$ is the recorded or estimated vertical component of particle velocity in the frequency f – wavenumber domain, $P(f, k_x, k_y)$ is the recorded pressure, and $\rho$ is the density in the recording medium.

The term $k_z$, which can be expressed as

$$[3] \quad k_z(f, k_x, k_y) = \sqrt{(2\pi f/c)^2 - k_x{}^2 - k_y{}^2} \quad ,$$

is the absolute value of the vertical wavenumber expressed
in terms of horizontal wavenumbers in the in-line direction
5   $k_x$ and the cross-line direction $k_y$, and the velocity c of
the recording medium.  It should be noted that the
decomposition could also be achieved by computing the up-
going component of the recorded pressure $P^-$ using equations
[1], leading expression which include terms of $1/k_z$.  Such
10  terms can be approximated using similar expansions as
described below.

In known decomposition methods using any of the above
equations [1,2], the cross-line or y-directions is mostly
15  ignored or a radial symmetry is assumed, with the vertical
wavenumber then being computed using an approximation based
exclusively on a one-dimensional direction, i.e. the in-line
wavenumber $k_x$ or the radial wavenumber $k_r$. When 3D effects
of the sub-surface or acquisition geometry are significant,
20  such approximations are no longer valid.

Equations as those described herein can be implemented in
the common mid-point domain, which is proposed by Vermeer
(1994) and Thomas (2000). It is however fruitful to rewrite
25  or approximate equation [3] into a form constituting a
cascade (sum or product) of one-dimensional (1D) spatial
filters acting in the x- or y-directions only.  This
represents a computationally attractive way of filtering the
data (both in terms of CPU and resorting data between
30  different domains).  One way to obtain filters of this form
is to make suitable Taylor expansions of the horizontal
wavenumbers in the square-root term around zero wavenumbers.

12

This approximation remains valid for data corresponding to propagating waves at $k_x^2 + k_y^2 < (2\pi f/c)^2$.

The expression for the vertical wavenumber can be rewritten and expanded in $k_x$ and $k_y$ to produce a few different alternative expansions that can be implemented using a cascade of filters that only act in the cross-line or in-line direction one at a time:

[4a]

$$k_z(f, k_x, k_y) \approx$$

$$\sqrt{(2\pi f/c)^2 - k_x^2}\left(1 - \frac{k_y^2}{2\left((2\pi f/c)^2 - k_x^2\right)} - \frac{k_y^4}{8\left((2\pi f/c)^2 - k_x^2\right)^2} + O\left(k_x^6, k_y^6\right)\right).$$

[4b]

$$k_z(f, k_x, k_y) \approx$$

$$2\pi f/c\left(1 - \frac{k_x^2 + k_y^2}{2(2\pi f/c)^2} - \frac{k_x^4 + k_y^4 + 2k_x^2 k_y^2}{8(2\pi f/c)^4} + O\left(k_x^6, k_y^6\right)\right).$$

[4c]

$$k_z(f, k_x, k_y) \approx$$

$$\sqrt{(2\pi f/c)^2 - k_x^2} + \sqrt{(2\pi f/c)^2 - k_y^2} - 2\pi f \bigg/ c \left( 1 + \frac{2k_x^2 k_y^2}{8(2\pi f/c)^4} + O\!\left(k_x^6, k_y^6\right) \right)$$

Equations [4a-4c] represent different ways to proceed with an implementation of the filter with two spatial components

5      that only rely on being able to filter the data along two perpendicular spatial directions one at a time which is exactly what can be achieved using the method described above in a cross-line geometry.  Note that this does not mean that only a "cross" of mid-points are used in filtering

10     the data.  All cross-terms of multiplications of terms with different horizontal wavenumbers will result in a "virtual" carpet of data being used of dimension of the length of the spatial filters in both directions.

15     After the decomposition **22**, multiples could be removed **23** from the data set.  Further processing steps and/or filtering steps **24** could be performed on the decomposed data set. Using what is commonly referred to as imaging or migration **25** the data set can be further processed to yield

20     an image of subterranean layers.  These images are used for hydrocarbon exploration and reservoir characterization.  The optional steps **21** and **23** – **25** are indicated in FIG. 2 as dashed blocks.

25     FIG. 3 shows a panel of the exact wavefield decomposition filter using equation [3] in the top left, difference between equation [3] and the filter approximation [4a] in the top right, difference between equation [3] and the

14

filter approximation [4b] in the bottom left, and difference
between equation [3] and the filter approximation [4c] in
the bottom right.  Note that these plots only assess the
accuracy of the filter approximations and do not include the
5    error due to their discretization.  In other words, the
plots do not show inaccuracies related to how the different
terms in the spatial filter approximations [4a-4c] are
implemented (e.g., using 3-point or 5-point derivative
approximation).  This would of course introduce a dependence
10   on frequency as well.  However, this is of secondary
importance as appropriate approximations that are
sufficiently accurate are straightforward to find.

From the top right of FIG. 3, it can be seen that equation
15   [4a] which was obtained by making a Taylor expansion in the
y-direction only results in the best approximation of the
three examples for azimuths close to the in-line cable
direction.  Equation [4b] which is used in the difference
plot at the bottom left of FIG. 3 results in an
20   approximation which is equally good along all azimuths.  An
advantage with this filter is that it can be fully
implemented as a compact filter.  Equation [4c] which is
used in the difference plot at the bottom right of FIG. 3
results in a fully accurate approximation both along the in-
25   line and cross-line azimuths.  The filter can be implemented
using a compact filter approximation for the cross-term
only.  Exactly which of the alternative implementation [4a-
4c] that is most attractive may vary depending on different
combinations of requirements in terms of computational cost
30   (CPU and data access) and accuracy.

While the invention has been described in conjunction with
the exemplary embodiments described above, many equivalent
modifications and variations will be apparent to those
skilled in the art when given this disclosure. Accordingly,

5    the exemplary embodiments of the invention set forth above
are considered to be illustrative and not limiting.  Various
changes to the described embodiments may be made without
departing from the spirit and scope of the invention.

10   The above approximations or similar approximations can be
used for example with the separations developed by Amundsen
et al. in the above cited United States patent no. 6,101,408
or in: "Multiple attenuation and P/S splitting of
multicomponent OBC data at a heterogeneous sea floor", Wave

15   Motion 32 (2000), 67-78. In the latter document, demultiple
or decomposition equations are found for elastic
decomposition (particle velocity, traction) or P/S wave
splitting below the sea floor.